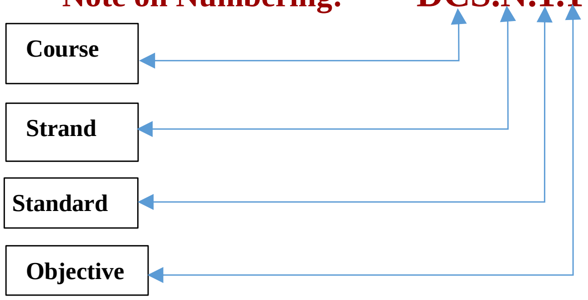
Discrete Mathematics for Computer Science Indicators

<Discrete Mathematics for Computer Science> *(Draft 3 Indicators)*
<NCDPI State Course Code> *(if applicable)*
Note on Numbering: DCS.N.1.1



Discrete Math for Computer Science (Abbreviation DCS)

DCS Course Description: The purpose of this course is to introduce discrete structures that are the backbone of computer science. Discrete mathematics is the study of mathematical structures that are countable or otherwise distinct and separable. The mathematics of modern computer science is built almost entirely on discrete mathematics, such as logic, combinatorics, proof, and graph theory. At most universities, an undergraduate-level course in discrete mathematics is required for students who plan to pursue careers as computer programmers, software engineers, data scientists, security analysts and financial analysts. Students will be prepared for college level algebra, statistics, and discrete mathematics courses.

Approximate Time Needed: _______ class periods	**Each class period lasts about 90 minutes or 55 minutes**

Standards for Mathematical Practice

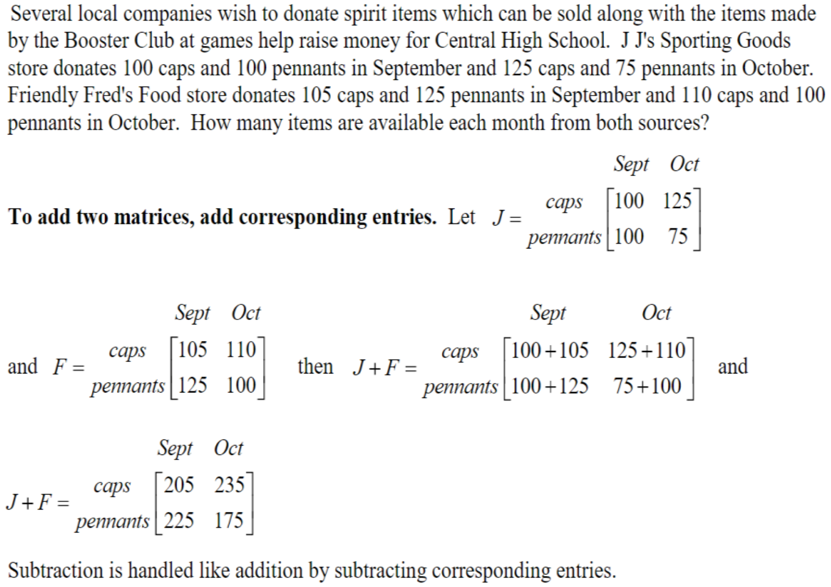
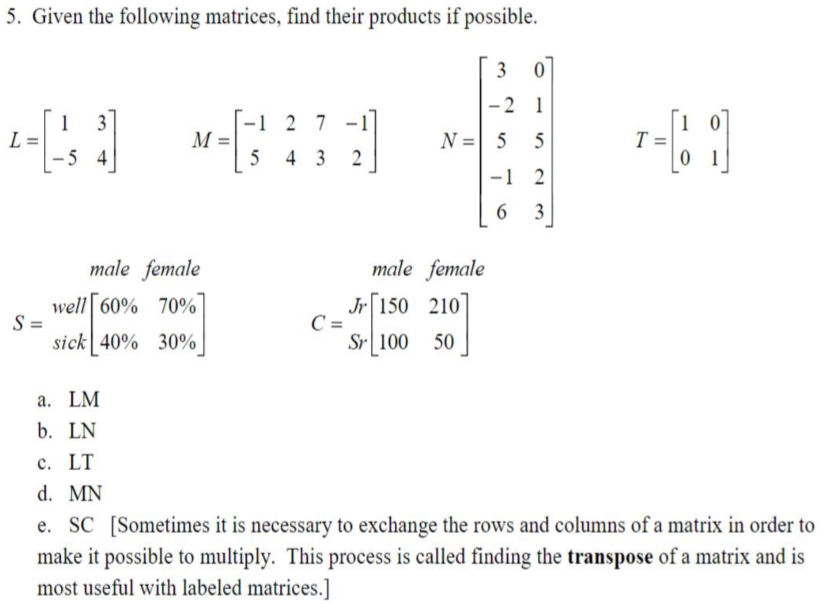
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.
9. Use strategies and procedures flexibly.
10. Reflect on mistakes and misconceptions.

Number and Quantity *(N)*

DCS.N.1 Apply operations with matrices and vectors.

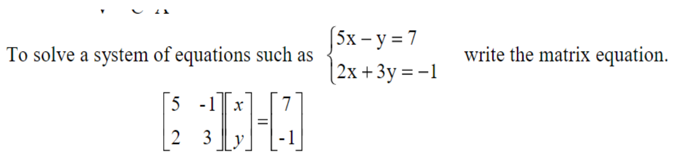
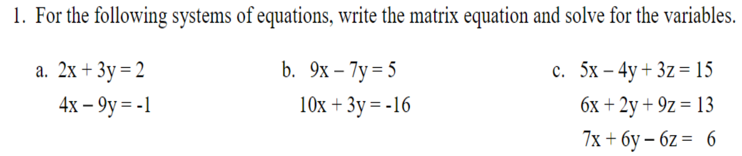
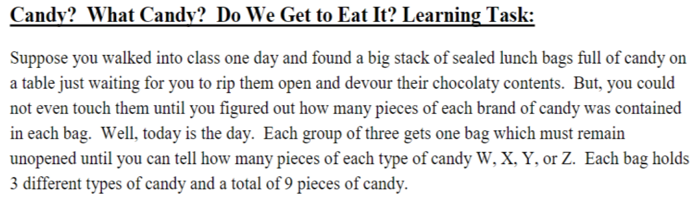
DCS.N.1.1 Implement procedures of addition, subtraction, multiplication, and scalar multiplication on matrices.	**Performance Indicator DCS.N.1.1** Let $M = \begin{bmatrix} -2 & 0 & 5 \\ 1 & -3 & 4 \end{bmatrix}$, then $3M =$ Solution: $3M = \begin{bmatrix} -6 & 0 & 15 \\ 3 & -9 & 12 \end{bmatrix}$.

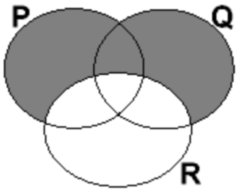
Discrete Mathematics for Computer Science Indicators

DCS.N.1.2 Implement procedures of addition, subtraction, and scalar multiplication on vectors.	**Performance Indicator DCS.N.1.2** ***(From Georgia StandadsM3:U1)***  Several local companies wish to donate spirit items which can be sold along with the items made by the Booster Club at games help raise money for Central High School. J J's Sporting Goods store donates 100 caps and 100 pennants in September and 125 caps and 75 pennants in October. Friendly Fred's Food store donates 105 caps and 125 pennants in September and 110 caps and 100 pennants in October. How many items are available each month from both sources? **To add two matrices, add corresponding entries.** Let $J = \begin{array}{ccc} & Sept & Oct \\ caps & 100 & 125 \\ pennants & 100 & 75 \end{array}$ and $F = \begin{array}{ccc} & Sept & Oct \\ caps & 105 & 110 \\ pennants & 125 & 100 \end{array}$ then $J+F = \begin{array}{ccc} & Sept & Oct \\ caps & 100+105 & 125+110 \\ pennants & 100+125 & 75+100 \end{array}$ and $J+F = \begin{array}{ccc} & Sept & Oct \\ caps & 205 & 235 \\ pennants & 225 & 175 \end{array}$ Subtraction is handled like addition by subtracting corresponding entries.  5. Given the following matrices, find their products if possible. $L = \begin{bmatrix} 1 & 3 \\ -5 & 4 \end{bmatrix}$ $M = \begin{bmatrix} -1 & 2 & 7 & -1 \\ 5 & 4 & 3 & 2 \end{bmatrix}$ $N = \begin{bmatrix} 3 & 0 \\ -2 & 1 \\ 5 & 5 \\ -1 & 2 \\ 6 & 3 \end{bmatrix}$ $T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $S = \begin{array}{ccc} & male & female \\ well & 60\% & 70\% \\ sick & 40\% & 30\% \end{array}$ $C = \begin{array}{ccc} & male & female \\ Jr & 150 & 210 \\ Sr & 100 & 50 \end{array}$ a. LM b. LN c. LT d. MN e. SC [Sometimes it is necessary to exchange the rows and columns of a matrix in order to make it possible to multiply. This process is called finding the **transpose** of a matrix and is most useful with labeled matrices.]
DCS.N.1.3 Implement procedures to find the inverse of a matrix.	**Performance Indicator DCS.N.1.3** Find the names of a famous piece of art by decoding the message 21, 0, 53, 2, 11, 3, 24 if the original coding matrix was $\begin{bmatrix} -3 & 5 \\ -1 & 2 \end{bmatrix}$

DCS.N.2 Understand matrices to solve problems.

DCS.N. 2.1 Organize data into matrices to solve problems.	**Performance Indicator DCS.N.2.1** 1. Central High School Booster Club Learning Task: (Edited from Georgia State Standards M3:U1) In order to raise money for the school, the Central High

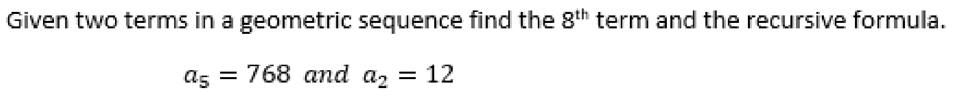
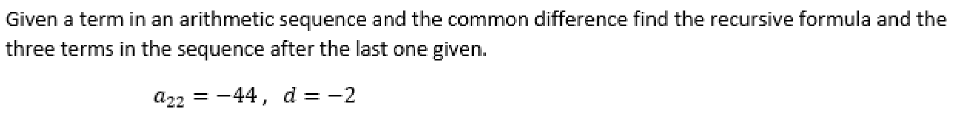
	School Booster Club offered spirit items prepared by members for sale at the school store and at games. They sold stuffed teddy bears dressed in school colors, tote bags and tee shirts with specially sewn and decorated School insignias. The teddy bears, tote bags, and tee shirts were purchased from wholesale suppliers and decorations were cut, sewn and painted, and attached to the items by booster club parents. The wholesale cost for each teddy bear was $4.00, each tote bag was $3.50 and each tee shirt was $3.25. Materials for the decorations cost $1.25 for the bears, $0.90 for the tote bags and $1.05 for the tee shirts. Show the wholesale cost of each item as well as the cost of decorations using a matrix. **Solution:** *Cost per Item* $A = \begin{array}{rc} & \begin{array}{ccc} \textit{bears} & \textit{totes} & \textit{shirts} \end{array} \\ \begin{array}{r} \textit{wholesale} \\ \textit{decorations} \end{array} & \begin{bmatrix} 4.00 & 3.50 & 3.25 \\ 1.25 & .90 & 1.05 \end{bmatrix} \end{array}$
DCS.N.2.2 Interpret solutions found using matrix operations including Leslie Models and Markov Chains, in context.	**Performance Indicator DCS.N.2.2** A video store owner has found that the probability that a customer who rented a movie today will rent a movie tomorrow is 35% while the probability that a customer who did not rent tomorrow is 10%. a. Write a transition matrix that represents this information. b. If 853 out of his 8745 customers rented a movie on Monday night (7892 of his customers didn't rent one on Monday) about how many customers can he expect to rent a movie on Tuesday? About how many of his customers can be expect to rent a movie three weeks from Monday?
DCS.N.2.3 Represent a system of linear equations as a matrix equation.	**Performance Indicator DCS.N.2.3**  To solve a system of equations such as $\begin{cases} 5x - y = 7 \\ 2x + 3y = -1 \end{cases}$ write the matrix equation. $\begin{bmatrix} 5 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$
DCS.N.2.4 Use inverse matrices to solve a system of linear equations with technology.	**Performance Indicator DCS.N.2.4**  1. For the following systems of equations, write the matrix equation and solve for the variables. a. $2x + 3y = 2$, $4x - 9y = -1$ b. $9x - 7y = 5$, $10x + 3y = -16$ c. $5x - 4y + 3z = 15$, $6x + 2y + 9z = 13$, $7x + 6y - 6z = 6$  **Candy? What Candy? Do We Get to Eat It? Learning Task:** Suppose you walked into class one day and found a big stack of sealed lunch bags full of candy on a table just waiting for you to rip them open and devour their chocolaty contents. But, you could not even touch them until you figured out how many pieces of each brand of candy was contained in each bag. Well, today is the day. Each group of three gets one bag which must remain unopened until you can tell how many pieces of each type of candy W, X, Y, or Z. Each bag holds 3 different types of candy and a total of 9 pieces of candy.
DCS.N.3 Understand set theory to solve problems.	
DCS.N.3.1 Recognize sets, subsets, and proper subsets.	**Performance Indicator DCS.N.3.1** Let B = {n l n is an integer and $n^2 < 20$}. Describe B using the listing method. Solution: {-4, -3, -2, -1, 0, 1, 2, 3, 4}

	List the subsets of {a, b, c}. Solution: ∅, {a}, {b}, {c}, {a,b}, {b,c}, {a,c}, {a,b,c} List the subsets of {a, b, c, d}. Solution: ∅, {a}, {b}, {c}, {d}, {a,b}, {b,c}, {a,c}, {a,d}, {b,d}, {c,d}, {a,b,c}, {a,b,d}, {a,c,d}, {b,c,d}, {a,b,c,d}
DCS.N.3.2 Implement set operations to find unions, intersections, complements and set differences with multiple sets.	**Performance Indicator DCS.N.3.2** Let U = {1,2,3,4,5,6,7,8,9,10}, A = {2,4,6,8,10}, B = {1,3,5,7,9}, and C = {1,2,3,4}. Describe each of the following sets using the listing method. a) (A∪B) ∩ C, solution: {1,2,3,4} b) A∩B ∩C, solution: ∅ c) A^C, solution: {1,3,5,7,9} d) $(A\cup C)^C$, solution: {5,7,9} e) A - C, solution: {6,8,10}
DCS.N.3.3 Represent properties and relationships among sets using Venn diagrams.	**Performance Indicator DCS.N.3.3** Create and shade the portion of the Venn diagram that illustrates the following set: $(P \cup Q) \cap R^c$ Solution: 
DCS.N.3.4 Interpret Venn diagrams to solve problems.	**Performance Indicator DCS.N.3.4**
DCS.N. 4 Understand statements related to number theory and set theory.	
DCS.N.4.1 Use the Euclidean Algorithm to determine greatest common factor and least common multiple.	**Performance Indicator DCS.N.4.1** Find the greatest common factor of 18 and 27. **Solution:** gcf (18,27) = 9. Find the quotient and remainder when 32 is divided by 12. **Solution:** quotient = 2, remainder = 8, so 32 = 12 (2) + 8 Find the greatest common factor of 36 and 14 using the Euclidean Algorithm. **Solution:** gcf (36, 14) = 2 A skateboarder and a runner go around a loop in the same direction. The skateboarder completes the loop every 6 minutes, and the runner completes the loop every 8 minutes. If they start at the same time from the same location, how long will take before they are at the starting position at the same time?

	Solution: 24 minutes
DCS.N.4.2 Use the Fundamental Theorem of Arithmetic to solve problems.	**Performance Indicator DCS.N.4.2** Find the greatest common factor of $2^2 7^3$ and $2^2 5^3 7^2$. **Solution:** A factor of $2^2 7^3$ will have at most two factors of 2 and three factors of 7. A factor of $2^3 5^3 7^2$ will have at most three factors of 2, three factors of 5 and two factors of 7. Based on this information, there will be two factors of 2, zero factors of 5 and two factors of 7 in the greatest common factor (i.e., $2^2 7^2$).
DCS.N.4.3 Conclude that sets are equal using the properties of set operations.	**Performance Indicator DCS.N.4.3** Make a conjecture about the number of subsets of a set with five elements. **Solution:** 2^5 Make a conjecture about the number of subsets of a set with n elements. **Solution:** 2^n Provide a justification for the following: : If A and B are sets, then $(A \cup B)^C = A^C \cap B^C$ **Solution:**
DCS.N.4.4 Explain theorems related to greatest common factor, least common multiple, even numbers, odd numbers, prime numbers, and composite numbers.	**Performance Indicator DCS.N.4.4** Suppose x is an odd integer and y is an odd integer. Is x + y an even or odd integer? Provide a justification. **Solution:** Let's examine a few examples. 3+5=8, 1+1=2, -5+-3=-8. If we think about an odd number as an even number plus one more, then the sum of two odd numbers would be the sum of two even numbers plus two more, which is an even number. This could be illustrated with a drawing as well.

Functions *(F)*

DCS.F.1 Apply recursively-defined relationships to solve problems.

DCS.F.1.1 Implement procedures to find the *nth* term in an arithmetic or geometric sequence using spreadsheets.	**Performance Indicator DCS.F.1.1**  Given two terms in a geometric sequence find the 8th term and the recursive formula. $a_5 = 768 \;\; and \;\; a_2 = 12$  Given a term in an arithmetic sequence and the common difference find the recursive formula and the three terms in the sequence after the last one given. $a_{22} = -44\,, \;\; d = -2$
DCS.F.1.2 Represent the sum of a sequence using sigma notation.	**Performance Indicator DCS.F.1.2**
DCS.F.1.3 Implement procedures to find the sum of a finite sequence.	**Performance Indicator DCS.F.1.3**

	Find the sum of the given geometric or arithmetic series: a. $\sum_{n=1}^{11} 4\left(-\frac{1}{3}\right)^{n-1}$ b. $5+15+45+\cdots+98415$ c. $\sum_{n=1}^{12}(6-2n)$
DCS.F.1.4 Implement procedures to find the sum of an infinite sequence and determine if the series converges or diverges.	**Performance Indicator DCS.F.1.4** Find the sum of the series: a. $\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{n-1}$ b. $5+20+80+320+1280+\cdots$
DCS.F.1.5 Interpret the solutions to arithmetic and geometric sequences and series problems, in context.	**Performance Indicator DCS.F.1.5** Determine if the following series converge or diverge. If they do converge, find the sum: $\sum_{n=1}^{\infty} 3(0.75)^{n-1}$

Statistics and Probability *(SP)*

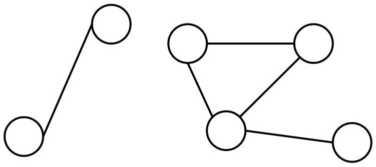
DCS.SP.1 Apply combinatorics concepts to solve problems.

DCS.SP.1.1 Implement the Fundamental Counting Principle to solve problems.	**Performance Indicator DCS.SP.1.1** You take a survey with five "yes" or "no" answers. How many different ways could you complete the survey? **Solution:** There are 2 choices for each question (Yes or No). So the total number of possible ways to answer is:

	2 * 2 * 2 * 2 * 2 = 32.
DCS.SP.1.2 Implement procedures to calculate a permutation or combination.	**Performance Indicator DCS.SP.1.2** In how many ways can a group of 5 members be formed by selecting 3 boys out of 6 and 2 girls out of 5 ? **Solution:**$C(6,3) \cdot C(5,2) = 20 \cdot 10 = 200$ How many more ways can 10 juniors running for the positions of president, vice president, secretary, and treasurer be selected when compared to 12 sophomores running for 5 identical positions of class representative? **Solution:** $P(10,4) - C(12,5) = 4248$

Graph Theory *(GT)*

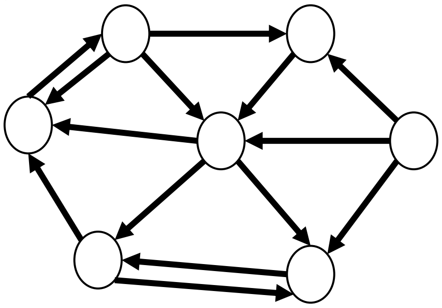
DCS.GT.1 Understand graph theory to model relationships and solve problems.

<table>
<tr>
<td>DCS.GT.1.1 Represent real world situations using a vertex-edge graph, adjacency matrix, and vertex-edge table.</td>
<td>Performance Indicator DCS.GT.1.1
The following table shows a list of students who are Facebook “friends.”
<table>
<tr><th>Student</th><th>Is friends with:</th></tr>
<tr><td>Anna</td><td>Beth</td></tr>
<tr><td>Chun</td><td>Dalton</td></tr>
<tr><td>Dalton</td><td>Edwin</td></tr>
<tr><td>Edwin</td><td>Chun</td></tr>
<tr><td>Edwin</td><td>Felicia</td></tr>
</table>
A) Create a vertex edge graph that represents the information in the table.
B) Construct an adjacency matrix to represent the information in the table.
C) Is the graph connected?
D) Is the graph complete?
E) What is the degree of Edwin’s vertex?
Solution:
A)

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$
B)
C) No
D) No
</td>
</tr>
</table>

<table>
<tr>
<td></td>
<td>E) 3</td>
</tr>
<tr>
<td>DCS.GT.1.2 Test graphs and digraphs for Euler paths, Euler circuits, Hamiltonian paths, or Hamiltonian circuits.</td>
<td>

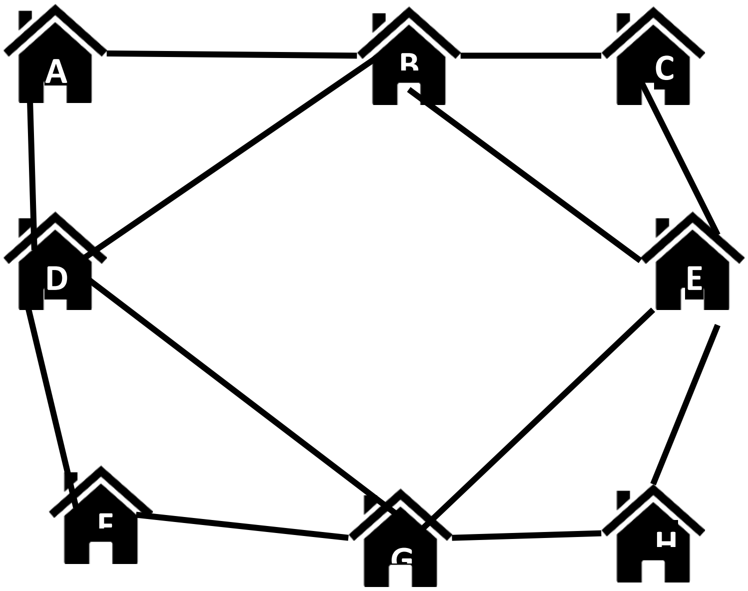
Performance Indicator DCS.GT.1.2

A) A group of 7 Discrete math students are working on a project and plan to share information via text messages. The digraph below shows the 7 students' communication outside of class. Is it possible to create a circuit, starting with one person, traveling to every other person exactly once and get back to the owner so he knows it went around?



Solution: No, a Hamiltonian Circuit does not exist. A path does exist, but must start with Edwin. One example is E - G - F - C - A - B – D

B) The graph below shows a small neighborhood of homes along with roads connecting them.



a. Is it possible for the person in house B to travel to each house to hand out flyers for her missing pet pig and return back home without passing by the same house twice? If so, name the circuit, and if not, explain.

Solution: Yes, it is possible. One example is B-C-E-H-G-F-D-A-B

b. The latest snow storm has dumped over a foot of snow. Is it possible for the person in House F to plow every road exactly once without traveling over the same road twice? If so, name the circuit and if not, explain.

</td>
</tr>
</table>

Solution: Yes, it is possible since every vertex has an even degree. One example is F- D- B- E- G - D - A - B - C - E - H - G - F

DCS.GT.1.3 Interpret a complete digraph to determine rank.

Performance Indicator DCS.GT.1.4

Create a complete digraph to represent the round robin tournament below:

Game	AB	AC	AD	BC	BD	CD
Winner	A	C	D	C	D	C

List all possible Hamiltonian paths.

Solution: There is only one possible path, C - D - A - B

DCS.GT.2 Apply graph theory to solve problems.

DCS.GT.2.1 Implement critical path analysis algorithms to determine the minimum project time.

Performance Indicator DCS.GT.2.1

Create a graph to represent the project listed below:

Task	Time	Prerequisite
A	3	none
B	4	none
C	2	A
D	3	B, C
E	5	D
F	2	B
G	1	E,F

a. What is the Earliest Start Time for Task G?
b. What is the Latest Start Time for Task B?
c. Name the Critical Path.
d. How much slack time is available for task F?
e. What is the minimum project time required?

Solution:

a. 13
b. 1
c. Start-A-C-D-E-G-Finish
d. 7
e. 14

DCS.GT.2.2 Implement the brute force method, the nearest-neighbor algorithm, and the cheapest-link algorithm to find solutions to a Traveling Salesperson Problem.

Performance Indicator DCS.GT.2.2

Kayana decided to do a driving tour of 4 college campuses to determine which school(s) she is most interested in attending. The table below shows the distances between each university.

	ASU	NCSU	UNCA	UNCW
ASU	--------	186	86.5	317
NCSU	186	--------	246	129
UNCA	86.5	246	--------	334
UNCW	317	129	334	--------

a. Draw a weighted graph using this data.
b. What is the total distance Kayana will travel using the Nearest Neighbor Algorithm, if she begins and ends her travel at Appalachian State University (ASU)?
c. What is the total distance Kayana will travel using the Cheapest Link algorithm, if she begins and ends at ASU?
d. What is the optimal solution? Draw a tree diagram to show all possible routes.

DCS.GT.2.3 Implement vertex-coloring techniques to solve problems.

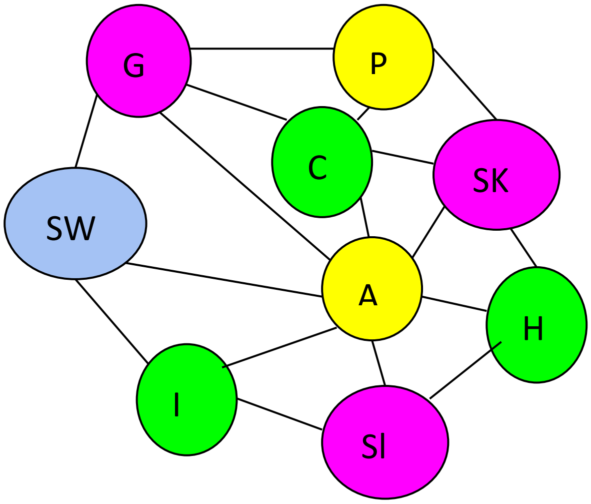
Performance Indicator DCS.GT.2.3

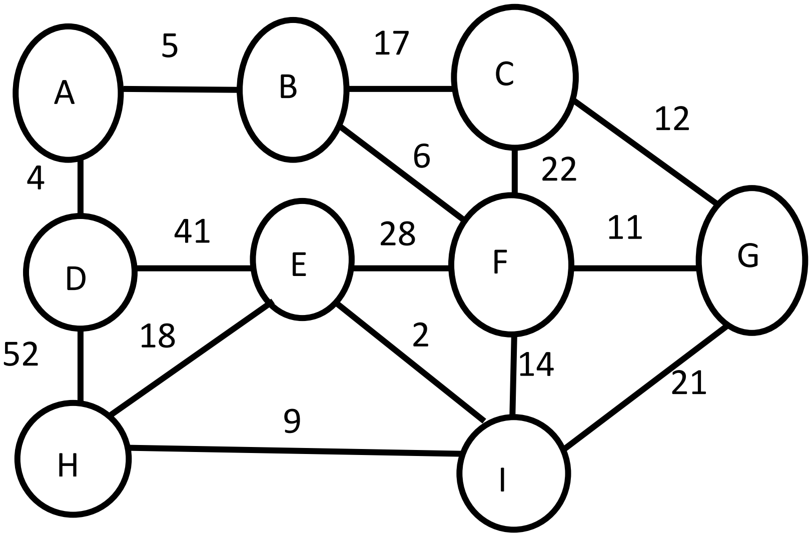
1. It is time for Mrs. Blevins to create a testing schedule for Discrete High School. The difficulty is that some students have multiple tested subjects, so they must be scheduled for different days. The table below shows which tests must be scheduled separately:

Test	Cannot be tested with
A	B, C, D, G
B	A, C, D, E
C	A, B, D, G, E
D	A, B, C, E, F
E	B, D
F	C, D
G	A, C

Draw a vertex-edge graph to represent this situation and find the chromatic number. What does the chromatic number mean in this situation?

2. Represent the following map as a vertex-edge graph. Use the graph to determine the chromatic number.

	 Solution:  Chromatic Number = 4
DCS.GT.2.4 Implement Kruskal and Prim's algorithms to determine the weight of the minimum spanning tree of a connected graph.	Performance Indicator: Twitter has 3 different types of advertisements, each at different costs. Given the following network of individuals and cost to share information among connected pairs, find the minimum cost to disseminate information to all individuals (minimum spanning tree):


A
B
C
5
17
12
4
6
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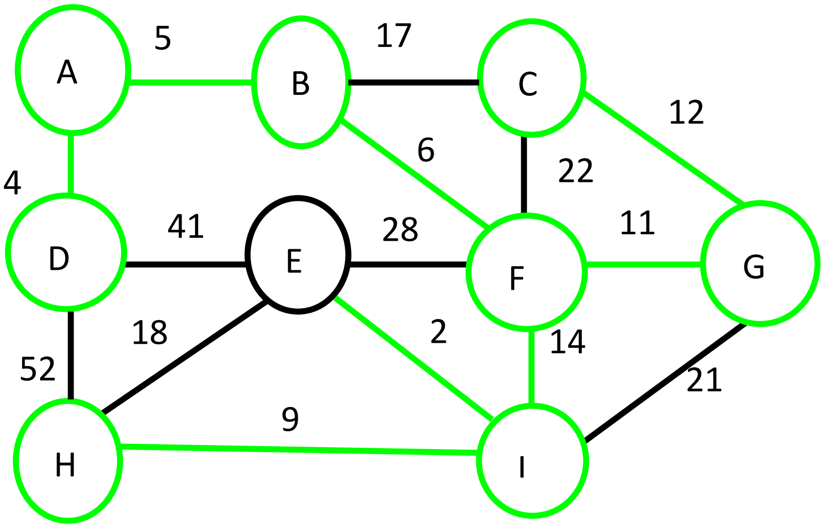
D
E
F
G
41
28
11
52
18
2
14
21
9
H
I

Solution:


A
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=63

Discrete Mathematics for Computer Science Indicators

Logic (*L*)

DCS.L.1 Evaluate mathematical logic to model and solve problems.	
DCS.L.1.1 Construct truth tables that encode the truth and falsity of two or more statements.	**Performance Indicator DCS.L.1.1** Construct a truth table for the statement $\neg P \wedge (P \rightarrow Q)$

P	Q	$\neg P$	$(P \rightarrow Q)$	$\neg P \wedge (P \rightarrow Q)$
T	T	F	T	F
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Discrete Mathematics for Computer Science Indicators

DCS.L.1.2 Critique logic arguments (e.g., determine if a statement is valid or whether an argument is a tautology or contradiction).

Performance Indicator DCS.L.1.2

1. Determine the validity of the following statement:

   If you invest in Corporation X, then you get rich.
   You didn't invest in Corporation X. Therefore, you didn't get rich.

Solution: Let p be the statement "You invest in Corporation X."
Let q be the statement "You get rich."
Then the argument has this symbolic form:

$$\begin{array}{l} p \to q \\ \sim p \\ \hline \therefore \sim q \end{array}$$

Interpret the truth table.
Notice that in the third row, the conclusion is FALSE while both premises are TRUE.
This tells us that the argument is INVALID.

		premise	premise	conclusion
p	q	$p \to q$	~p	~q
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Performance Indicator:

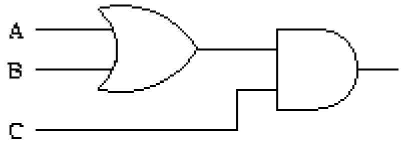
Show that the logical argument $(P \to Q) \vee (Q \to P)$ is a tautology.

P	Q	$P \to Q$	$Q \to P$	$(P \to Q) \vee (Q \to P)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

DCS.L.1.3 Check 1s and 0s to determine whether a statement is true or false using Boolean logic circuits.

Performance Indicator DCS.L.1.3

If A = 0 and B = 1 and C=0, find whether the following statement is true (1) or false (0):



Solution: 0 or 1 has an output of 1 and then 1 and 0 has an output of 0, so the final output is 0 (False).

DCS.L.1.4 Judge whether two statements are logically equivalent using truth tables.

Performance Indicator DCS.L.1.4

Use a truth table to prove $p \rightarrow q \equiv (q \vee \neg p)$

Solution:

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

P	Q	$\neg P$	$Q \vee \neg P$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

Since the all sets of inputs return the same outputs, respectively, these two statements are logically equivalent